LESSON 1 Units • Scientific Notation • Significant Digits

1.A units

Physics is a mental science and also is an experimental science. Everything we know about the physical world has been gained by careful mental consideration of our observations. We use experiments and measurements to allow us to make our thought processes more precise. When we measure a quantity, we compare it with some reference standard. The standard is called a *unit* of the quantity. Meters, centimeters, feet, and inches are units of distance. Kilograms and grams are units of mass. Seconds, minutes, and hours are units of time. Almost every country in the world except the United States uses a system of units called SI, which stands for *Système International d'Unités*, often called the *metric system*. In this system the **meter** (approximately 39.37 inches) is the base unit of length. The **kilogram** is the base unit of mass. The **second** is the base unit of time, and the **coulomb** is the base unit of charge. Most units are called ***derived units*** since they are defined by some combination of other base units. **The newton is an example. The *newton* is the unit of force in the metric system and is defined as a kilogram-meter per second squared.** We will discuss this in a later lesson. For now we will content ourselves with the knowledge that this book weighs about 20 newtons and an apple weighs about 2 newtons.

All versions of metric units are related to the base units, or derived units, by multiples of 10 or of one-tenth. The prefix *deca-* means "ten" and the prefix *deci-* means "tenth." The prefix *hecto-* (seldom used) means "hundred" and the prefix *centi-* means "hundredth." The prefix *kilo-* means "thousand" and the prefix *milli-* means "thousandth." The names of multiples of units are formed by attaching a unit to a prefix. Thus

1 kilometer equals 1000 meters.

1 centimeter equals $\frac{1}{100}$ meter.

The following list gives some examples of the use of the prefixes:

1 nanometer = 1 nm = 10^{-9} m	1 gram = 1 g = 10^{-3} kg
1 micrometer = 1 μm = 10^{-6} m	1 nanosecond = 1 ns = 10^{-9} s
1 millimeter = 1 mm = 10^{-3} m	1 microsecond = 1 μs = 10^{-6} s
1 centimeter = 1 cm = 10^{-2} m	1 millisecond = 1 ms = 10^{-3} s
1 kilometer = 1 km = 10^{3} m	1 minute = 1 min = 60 s
1 microgram = 1cc μg = 10^{-9} kg	1 hour = 1 hr = 3600 s
1 milligram = 1 mg = 10^{-6} kg	1 day = 1 da = 86,400 s

Instead of SI units, the United States still uses the foot, the inch, the mile, the pound, the bushel, the peck, the tablespoon, etc., as units of measure. This system was inherited from the English and is often called the *U.S. Customary System* of units. This system is unwieldy and causes unnecessary confusion. Even the English have changed to SI units. We will use SI units in most of the problems in this book. However, we will also use English units because so many of the problems that we confront in our daily lives are stated in English units so we will have to practice converting between the systems.

abbreviations You will find that physics is a fun course, and you will find that the concepts of physics are simple and straightforward. Unfortunately, some abbreviations and notations that are used are difficult for the beginner to decipher, and if you can't read it, you can't understand it. To overcome this difficulty, we will begin by writing out many units in full. For example, instead of writing $N \cdot m$ and expecting you to read it as newton-meter, we will write it out. Instead of writing $m \cdot s^{-2}$, we will write out meters per second per second or meters per second squared.

1.B scientific notation

In physics we often use very large numbers and very small numbers. We find that a convention called **scientific notation** is very helpful. A number written in scientific notation has two factors. The first factor is a decimal number with the decimal point immediately following the first digit. The second factor is a power of 10 that tells the **true location** of the decimal point in the number. A **positive exponent** in the power of 10 indicates that the true location of the decimal point is **to the right** of where it is written. For example:

$$4.16 \times 10^5 \quad \text{means} \quad 4\underbrace{16000}_{\text{5 places}}.$$

$$3.24 \times 10^7 \quad \text{means} \quad 3\underbrace{2400000}_{\text{7 places}}.$$

If a power of 10 has a **negative exponent**, the true location of the decimal point is **to the left** of where it is written.

$$4.16 \times 10^{-5} \quad \text{means} \quad 0.\underbrace{00004}_{\text{5 places}}16$$

$$3.24 \times 10^{-7} \quad \text{means} \quad 0.\underbrace{0000003}_{\text{7 places}}24$$

We multiply numbers written in scientific notation by multiplying the first factors and adding the exponents in the powers of 10.

$$(4 \times 10^5)(2 \times 10^{-8}) = (4 \times 2) \times 10^{5-8} = 8 \times 10^{-3}$$

We divide numbers written in scientific notation by dividing the first factors and subtracting the exponent of the power of 10 on the bottom from the power of 10 on the top.

$$\frac{8 \times 10^{-4}}{2 \times 10^{-1}} = \frac{8}{2} \times 10^{-4-(-1)} = 4 \times 10^{-3}$$

example 1.1 Write these numbers in standard form:

(a) 4.16×10^{-13} (b) 4.16×10^5

solution (a) The 10^{-13} factor tells us to move the decimal point 13 places to the left. We do this and get

$$4.16 \times 10^{-13} = \mathbf{0.000000000000416}$$

(b) The 10^5 factor tells us to move the decimal point 5 places to the right. We do this and get

$$4.16 \times 10^5 = \mathbf{416{,}000}$$

example 1.2 Write these numbers in scientific notation:

(a) 0.000032 (b) 7,150,000

(c) 4000×10^{-10} (d) 7642×10^5

This answer does not have the decimal point just to the right of the first nonzero digit, so we need one more step. The factor 10^{-15} tells us that the decimal point is really 15 places to the left of where it is written. If we move it 1 place to the left, we still have 14 places to go. Thus we have

$$16 \times 10^{-15} \quad \text{equals} \quad (1.6 \times 10^{+1})10^{-15} \quad \text{equals} \quad \mathbf{1.6 \times 10^{-14}}$$

Facility with scientific notation comes through long-term practice. The problem sets will contain scientific notation problems for a few lessons. If you still have trouble, ask your teacher to provide additional practice problems.

reading a number

We read a decimal number by reading the whole number in front of the decimal point. Then we read the decimal point by saying **and.** Then we read the number to the right of the decimal point and append to this **the place value of the last digit.**

NUMBER	READ AS
320.413	three hundred twenty **and** four hundred thirteen thousandths
17.041,301	seventeen **and** forty-one thousand, three hundred one millionths

If there are no digits other than zero in front of the decimal point, we just read the number following the decimal point and name the place value of the last digit.

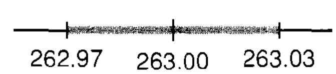
0.0412 four hundred twelve ten-thousandths

1.C significant digits

In mathematics we work with numbers, and numbers are exact. The number 263 has a value of exactly two hundred sixty-three. In science we use instruments to make measurements, and no measurement is exact. Every measurement is an approximation. **When we use measurements in calculations, we must be careful not to assume that the answer we get is more precise than is warranted by the preciseness of the measurements used to make the calculations.** We can state the range of uncertainty of a measurement by using a plus or minus notation. The measurement

$$263.00 \text{ cm} \pm 0.03 \text{ cm}$$

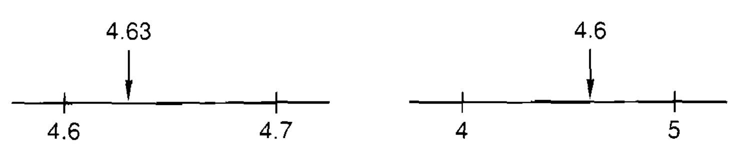
tells us that the range of uncertainty is 0.06 cm and extends 0.03 cm on either side of 263.00 cm.



This notation is satisfactory for giving the range of uncertainty of one measurement but is not helpful in determining the accuracy of a calculation in which more than one measurement is used. For this purpose we use a convention called **significant digits** or **significant figures.** The rules for significant digits are different for numbers whose last significant digit is not zero than for those whose last significant digit is zero.

nonzero last digits

To make our measurements as precise as possible, we usually include a last digit that is a guess. Consider the reading on the scale on the left. The arrow is between 4.6 and 4.65 and appears to be closer to 4.65, so we **guess** the last digit is about 3 and read 4.63.



The arrow on the scale on the right is between 4 and 5 and appears to be just a little closer to 5, so we guess the last digit is 6 and read 4.6.

If we do not write a measurement in scientific notation, it is necessary to use several rules to determine the number of significant digits.

Rules For Significant Digits

1. All nonzero digits are significant.
2. All interior zeros (zeros between nonzero digits) are significant.
3. Trailing zeros after the decimal point are significant. Otherwise there would be no reason to write them.
4. If the absolute value of a measurement is greater than zero and less than 1, the zeros immediately to the right of the decimal point are not significant.

$$\underbrace{0.0000}_{\text{not significant}}460 \rightarrow 4.60 \times 10^{-4}$$

We have covered all the rules but one. What do we do with whole number measurements that have terminal zeros? Does a measurement of 400 meters have a range of uncertainty of 100 meters or 10 meters or 1 meter? In this book we will use a precision bar to indicate the precision of whole number measurements with terminal zeros.

		Uncertainty
40,000	one significant digit	±10,000
$4\bar{0},000$	two significant digits	±1000
$40,\bar{0}00$	three significant digits	±100
$40,0\bar{0}0$	four significant digits	±10
$40,00\bar{0}$	five significant digits	±1

If the precision bar is not used, we suggest you consider that the number is a gross approximation and that none of the zeros is significant. We will use this convention in working the problems.

problem set 1

Write these numbers in scientific notation:

1. 0.0000652 **2.** 476.32

3. 475,000,000 **4.** 0.000376

5. Five hundred ten millionths

6. Seven hundred forty-two hundred-millionths

State the number of significant digits in each of these measurements:

7. 26.2300 cm **8.** 56.001 m^2 **9.** 3600 mi

10. 9000.02 $in.^3$ **11.** $26.0\bar{0}0$ km **12.** 0.000416 gallon

Add the following pairs of numbers and state your answer in scientific notation:

13. $(5.1 \times 10^7) + (2.3 \times 10^6)$ **14.** $(4.506 \times 10^{-4}) + (2.022 \times 10^{-2})$

Simplify and write your answer to one significant digit:

15. $\dfrac{(0.00077 \times 10^{-3})(40 \times 10^6)}{(0.00011 \times 10^5)(140,000)}$ **16.** $\dfrac{(3000 \times 10^{-14})(0.00008)}{(0.0002 \times 10^5)(200,000)}$

17. $\dfrac{(0.0006 \times 10^{-42})(2000 \times 10^{-4})}{0.004 \times 10^{-13}}$ **18.** $\dfrac{(0.0035 \times 10^{15})(0.002 \times 10^{17})}{7000 \times 10^{33}}$

19. $\dfrac{(0.0007 \times 10^{-23})(4000 \times 10^6)}{(0.00004)(7,000,000)}$ **20.** $\dfrac{(0.00056 \times 10^4)(7 \times 10^3)}{(0.00049 \times 10^{16})(0.00002 \times 10^{-5})}$