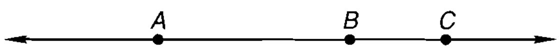
LESSON A Geometry review • Angles • Review of absolute value • Properties and definitions

A.A geometry review

Some fundamental mathematical terms are impossible to define exactly. We call these terms **primitive terms** or **undefined terms**. We define these terms as best we can and then use them to define other terms. The words **point**, **curve**, **line**, and **plane** are primitive terms.

A **point** is a location. When we put a dot on a piece of paper to mark a location, the dot is not the point because a mathematical point has no size and the dot does have size. We say that the dot is the **graph** of the mathematical point and marks the location of the point. A **curve** is an unbroken connection of points. Since points have no size, they cannot really be connected. Thus, we prefer to say that a curve defines the path traveled by a moving point. We can use a pencil to graph a curve. These figures are curves.

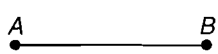
A mathematical **line** is a straight curve that has no ends. **Only one mathematical line can be drawn that passes through two designated points.** Since a line defines the path of a moving point that has no width, a line has no width. The pencil line that we draw marks the location of the mathematical line. When we use a pencil to draw the graph of a mathematical line, we often put arrowheads on the ends of the pencil line to emphasize that the mathematical line has no ends.



We can name a line by naming any two points on the line in any order. The line above can be called line *AB*, line *BA*, line *AC*, line *CA*, line *BC*, or line *CB*. Instead of writing the word "line," we can put a bar with two arrowheads above the letters, as we show here.

$$\overleftrightarrow{AB} \quad \overleftrightarrow{BA} \quad \overleftrightarrow{AC} \quad \overleftrightarrow{CA} \quad \overleftrightarrow{BC} \quad \overleftrightarrow{CB}$$

These notations are read as "line *AB*," "line *BA*," etc. We remember that a part of a line is called a **line segment** or just a **segment.** A segment has two endpoints. A segment can be named by naming the two endpoints in any order. The following segment can be called segment *AB* or segment *BA*.



Instead of writing the word "segment," we can draw a bar that has no arrowheads above the letters. Segment *AB* and segment *BA* can be written as

$$\overline{AB} \quad \text{and} \quad \overline{BA}$$

If we write the letters without using the bar, we are designating the length of the segment. If segment *AB* has a length of 2 centimeters, we could write either

13. $-3(-2 - 3 + 6) - [-5(-2) + 3(-2 - 4)]$

14. $-2 - 2^2 - 2^3 - 2^4$

15. $|-2| - |-4 - 2| + |8|$

16. $-|-3(2) - 3| - 2^2$

17. $-2^2 - 2^3 - |-2| - 2$

18. $-3[-1 - 2(-1 - 1)][-3(-2) - 1]$

19. $-3[-3(-4 - 1) - (-3 - 4)]$

20. $-2[(-3 + 1) - (-2 - 2)(-1 + 3)]$

21. $-2[-2(-4) - 2^3](-|2|)$

22. $-8 - 3^2 - (-2)^2 - 3(-2) + 2$

23. $-\{-[-5(-3 + 2)7]\}$

24. $-5 - |-3 - 4| - (3)^2 - 3$

25. $3(-2 + 5) - 2^2(2 - 3) - |-2|$

26. $\dfrac{-5 - (-2) + 8 - 4(5)}{6 - 4(-3)}$

27. $(-2)[|-3 - 4 - 5| - 2^3 - (-1)]$

28. $\dfrac{-3 - (-2) + 9 - (-5)}{7(|-3 + 4|)}$

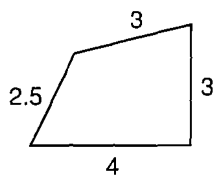
29. $4(-2)[-(7 - 3)(5 - 2)2]$

30. $4 - (-4) - 5(3 - 1) + 3(4)(-2)^3$

LESSON B Perimeter • Area • Volume • Surface area • Sectors of circles

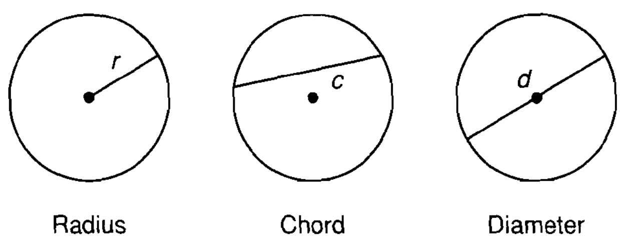
B.A perimeter

The **perimeter** of a closed, planar geometric figure is the distance around the figure. The perimeter of this figure is 12.5 units.



$$\text{Perimeter} = 2.5 + 3 + 3 + 4 = 12.5$$

We call the perimeter of a circle the **circumference** of the circle. The **radius** of a circle is the distance from the center of the circle to any point on the circle. A **chord** of a circle is a line segment whose endpoints are on the circle. A **diameter** is a chord that passes through the center of the circle. The length of a diameter is twice the length of a radius.



It takes about 3.14 diameters to go all the way around a circle. The exact number is a number we call *pi*. We use the symbol π to represent this number. It takes π diameters to equal the circumference, and it takes 2π radii to equal the circumference. When we use 3.14 as an approximation for π, we use the symbol $\approx$, which means "approximately equal to."

14. The complement of an angle is 10°. What is the measure of the angle?

15. The supplement of an angle is 60°. What is the measure of the angle?

Simplify:

16. $-2^2 - 2^3 - (-2)^2 - 2$

17. $-2^2 - |-4| + |4|$

18. $-|-3| - 3 - 3^2$

19. $-4 - (-3)^3 - 2^2 + |-4|$

20. $-3^2 - 2(-4 + 6)$

21. $-4(-2^2 - 3) - 5 + |-3|$

22. $-2[-1 - (-5)] - [-6(-2) + 3]$

23. $-2^2 - 2^3 - 2 - |-2|$

24. $-2 - |-3 - 4 + 8| - 2^2$

25. $-|-2 - 3 - 4| - |-2|$

26. $\dfrac{-5 - (-2) + 8 - 4(5) - 3}{6 - 4(-3)}$

27. $(-2)[|-3 + 4 - 5| - 2^3 - (-1)]$

28. $\dfrac{-|-5| - (-2) + 6 - 4(3 - |6 - 9|)}{5 - |(4)(-3)|}$

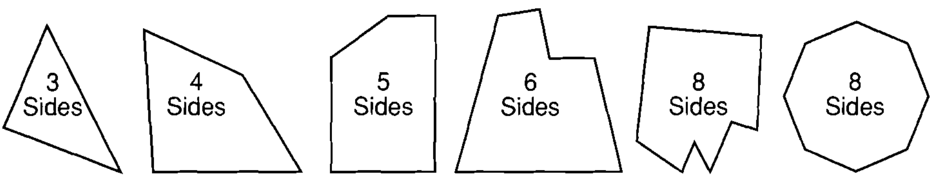
29. $\dfrac{-2 - (-3 - 2) - (-2 + 5)}{-4(2^2 - 3)(-2)}$

30. $-2(-3 + 4 - 6) - 2^2(-2) - 3(-2) - |-5|$

LESSON 1 *Polygons • Triangles • Transversals • Proportional segments*

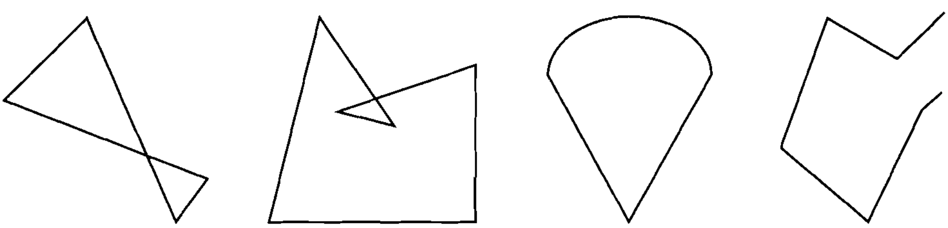
1.A polygons

Definitions often change. The definition of a polygon is a good example. The word is formed from the Greek roots *poly*, which means "more than one" or "many," and *gonon*, which means "angle." Thus, polygon literally means "more than one angle." In 1571 Diggs said that "Polygona are such figures that haue moe than foure sides." In 1656 Blount said that a polygon was a geometrical figure that "hath many corners."



Triangle Quadrilateral Pentagon Hexagon Octagon Octagon

All of the figures shown here fit Blount's definition of a polygon, but the two on the left do not have enough sides for Diggs's definition. Modern authors tend to define polygons as simple, closed, flat geometric figures whose sides are straight lines. The figures below are not polygons.



(a) (b) (c) (d)

The sides of the figures (a) and (b) cross, so these are not simple, closed geometric figures. One "side" of (c) is not a straight line, and figure (d) is not a closed figure. The five figures

14. The complement of an angle is 10°. What is the measure of the angle?

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Simplify:

16. $-2^2 - 2^3 - (-2)^2 - 2$

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LESSON 1 *Polygons • Triangles • Transversals • Proportional segments*

1.A polygons

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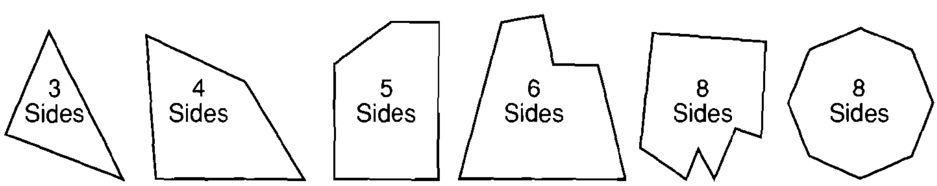

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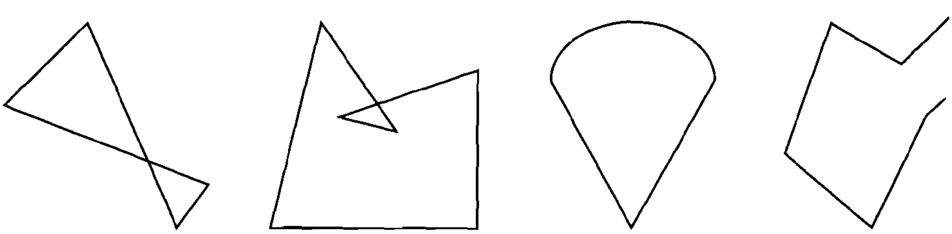
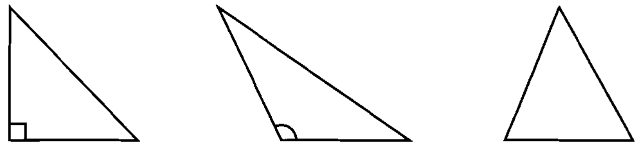


(a) (b) (c) (d)

The sides of the figures (a) and (b) cross, so these are not simple, closed geometric figures. One "side" of (c) is not a straight line, and figure (d) is not a closed figure. The five figures

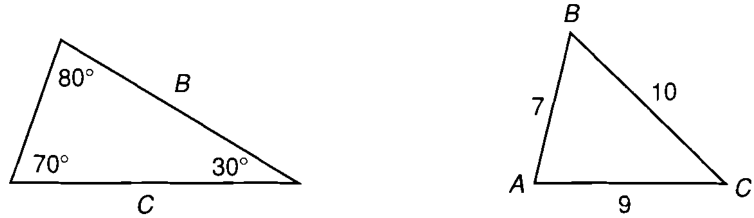
1.B triangles

If a triangle has a right angle, the triangle is a **right triangle.** If one angle is greater than 90°, the triangle is an **obtuse triangle.** If all angles are less than 90°, the triangle is an **acute triangle.**



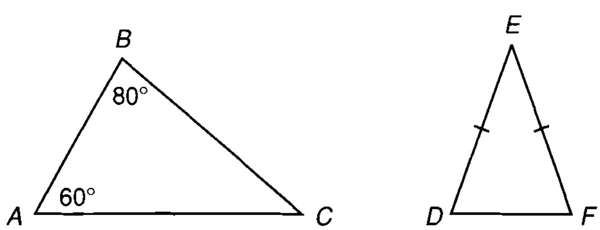
Right triangle   Obtuse triangle   Acute triangle

The sum of the measures of the three angles of any triangle is 180°. The greatest angle is opposite the longest side, and the smallest angle is opposite the shortest side. In the triangle on the left below, we know that the length of side C is greater than the length of side B because 80° is greater than 70°.



In the triangle on the right, we know that C is the smallest angle because it is opposite the shortest side. Angle A is the largest angle because it is opposite the longest side.

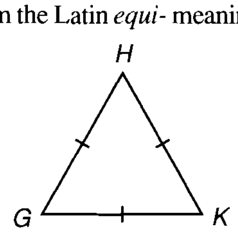
In a triangle, the angles opposite sides of equal lengths have equal measures. The sides opposite angles of equal measures have equal lengths.



Scalene triangle   Isosceles triangle

Angle C in the figure on the left must be a 40° angle, because the sum of all three angles must be 180°. **In the same figure all the angles have different measures, so all the sides must have different lengths.** If all the sides of a triangle have different lengths, the triangle is called a **scalene triangle.** The identical tick marks on two sides of the triangle on the right above tell us that these two sides have equal lengths. Thus, angle D and angle F must have equal measures. A triangle that has at least two equal sides (and two equal angles) is called an **isosceles triangle.** The word **isosceles** derives from the Greek prefix *iso-*, meaning "equal," and the Greek word *skelos*, meaning "leg."

The triangle shown below has three sides whose lengths are equal, so we call this triangle an **equilateral triangle**, from the Latin *equi-* meaning "equal" and *latus* meaning "side."



Equilateral triangle

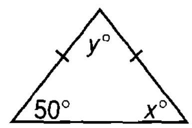
The identical tick marks indicate that the lengths of the sides are equal. Since angles opposite equal sides have equal measures, all three angles in this triangle must have equal measures.

Each angle must have a measure of 60° because $3 \times 60°$ equals 180°. Since an equilateral triangle has at least two sides whose lengths are equal, an equilateral triangle is also an isosceles triangle. We summarize these very important properties of triangles in the following boxes.

> If two sides of a triangle have equal lengths, the angles opposite these sides have equal measures. If two angles of a triangle have equal measures, the sides opposite these angles have equal lengths.

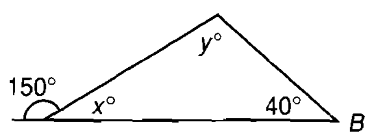
> When the three sides of a triangle have equal lengths, all three angles are 60° angles. If the three angles of a triangle are equal, they must be 60° angles and the three sides must have equal lengths.

example 1.1 Find x and y.



solution **In any triangle the angles opposite equal sides are equal angles.** Thus x is 50 and angle x is a **50° angle.** The sum of three angles in a triangle is 180°, so y must be 80 and angle y is an **80° angle.**

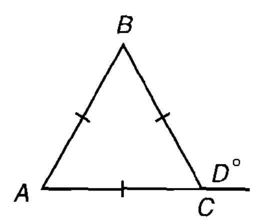
example 1.2 Find x and y.



solution The 150° angle and angle x form a 180° angle. Thus, angle x is a **30° angle.** Since angle B is a 40° angle, angle y must be a **110° angle** so that the sum of the angles will be 180°. We check by adding all three angles.

$$30 + 40 + 110 = 180 \qquad \text{check}$$

example 1.3 This triangle is an equilateral triangle. Find D.

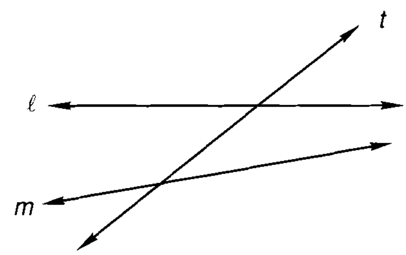


solution If the triangle is an equilateral triangle, all three angles are equal and each angle is a 60° angle. Angle D and one of the 60° angles form a straight angle. Thus, angle D is a 120° angle.

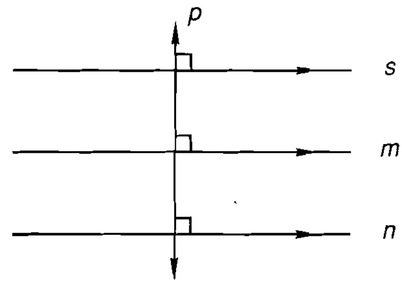
$$m\angle D = \mathbf{120°}$$

1.C transversals

A **transversal** is a line that cuts or intersects two or more other lines. **If a transversal intersects two or more lines that are parallel and if the transversal is perpendicular to one of the parallel lines, it is perpendicular to all the parallel lines.** We use the symbol $\parallel$ to mean parallel and $\nparallel$ to mean not parallel.



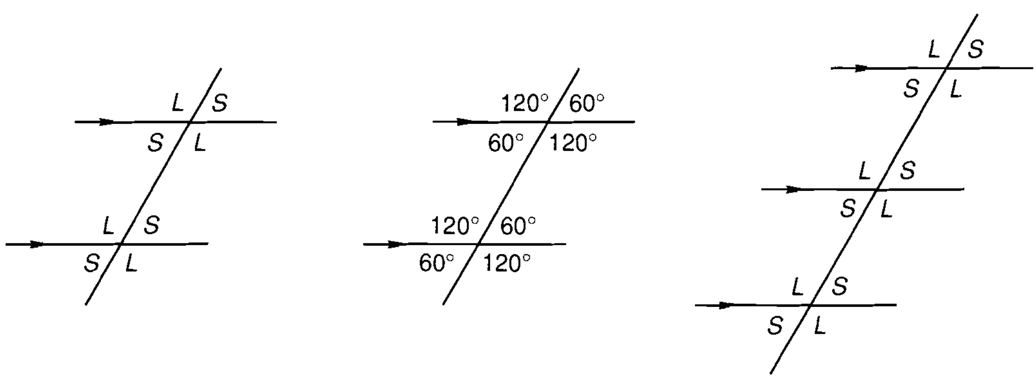
ℓ ∦ m



s ∥ m ∥ n

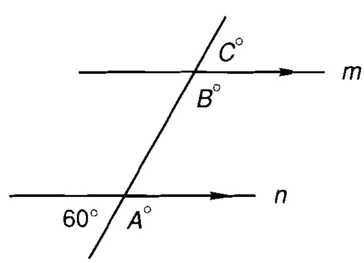
In the left-hand figure, line t is a transversal because it intersects both line m and line ℓ. In the right-hand figure, line p is a transversal because it intersects lines s, m, and n. These lines are parallel lines, as indicated by the arrowheads that are not on the ends of the segments. Because the transversal p is perpendicular to one of the parallel lines, it is perpendicular to all the parallel lines. We omit the arrowheads on the ends of these lines because the arrowheads would clutter the diagram.

If the transversal is not perpendicular to the lines, two groups of equal angles are formed. **Half the angles are "large angles" that are equal angles and are greater than 90°. Half the angles are "small angles" that are equal angles and are less than 90°.**



On the left we use the letters L and S to mean "large" and "small." We note that together a large angle and a small angle form a straight angle (180°), so the large angles and the small angles are supplementary angles. In the center figure, the large angles are 120° angles and the small angles are 60° angles. Thus, the sum of any large angle and any small angle is 180°. In the figure on the right, we see that when a transversal cuts three parallel lines, six equal large angles are formed and six equal small angles are formed.

example 1.4 Find A, B, and C.



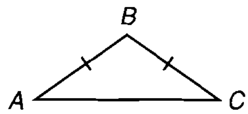
solution Angle A and the 60° angle form a straight angle, which measures 180°. Thus, A is **120.** Lines m and n are parallel, so all the small angles are equal and all the large angles are equal. Thus, C equals **60** and B equals **120.**

1.D proportional segments

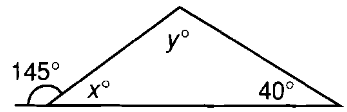
When three or more parallel lines are cut by two transversals, the lengths of the corresponding segments of the transversals are proportional. This means that the lengths of the segments of one transversal are related to the lengths of the corresponding segments of the other transversal by a number called the **scale factor.**

practice

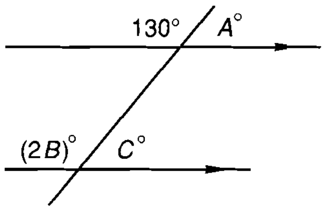
a. $m\angle A = 35°$. Find $m\angle C$ and $m\angle B$.



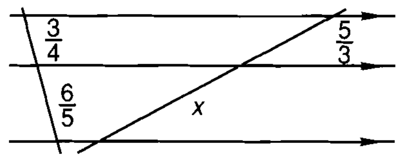
b. Find x and y.



c. Find A, B, and C.

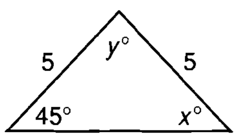


d. Find x.

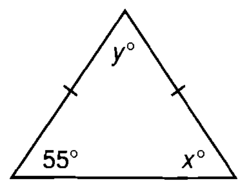


problem set 1

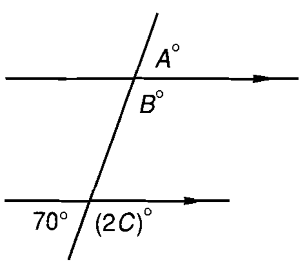
1. Find x and y.



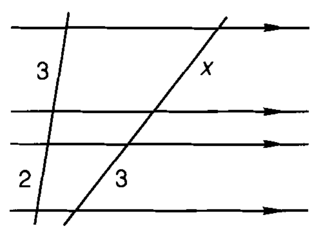
2. Find x and y.



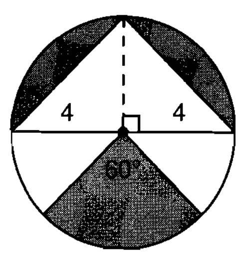
3. Find A, B, and C.



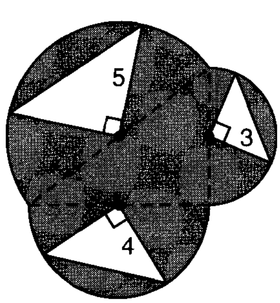
4. Find x.



5. The diameter of the circle is 8 cm, as shown. The sum of the top two shaded areas is the area of half the circle minus the area of the triangle. The shaded area below is a 60° sector of the circle. Find the sum of the three shaded areas.



6. Find the area of the shaded portion of this figure. Dimensions are in centimeters.



7. Find the perimeter of the figure. All angles that look like right angles are right angles. Dimensions are in feet.